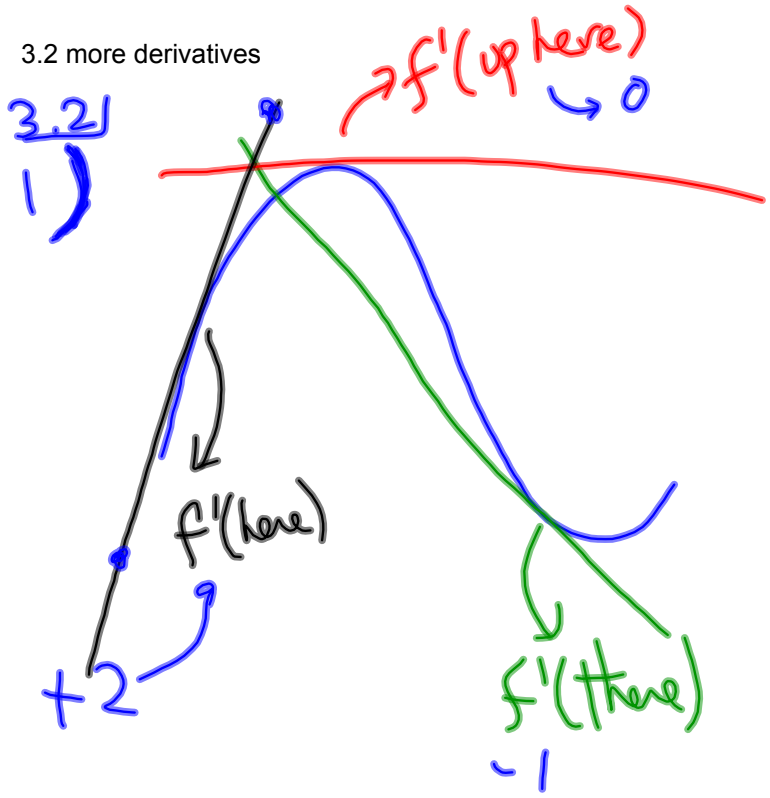
3.2 more derivatives

2014-10-07 day 30

3.2

1)



instantaneous rate of change

value of a derivative at a point

slope of tangent line

3.2 more derivatives

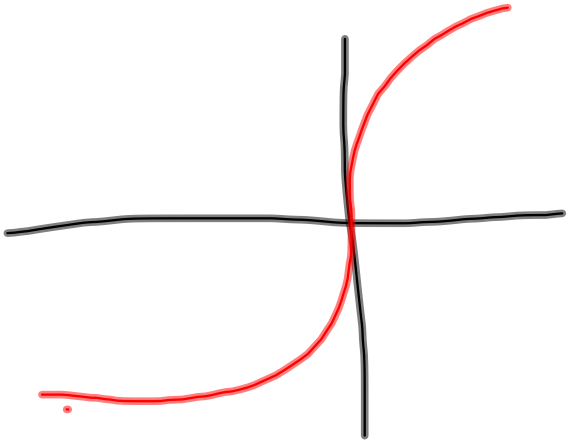
2014-10-07 day 30

slope of a vertical line is undefined.

So

value of derivative at that x does not exist.

Look at $x^{(1/3)}$.



3.2 more derivatives 2014-10-07 day 30

1) a) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^2-3x^2}{h}$

$= \lim_{h\to 0} \frac{3(x^2+2xh+h^2)-3x^2}{h}$ $(x+h)(x+h)$

$= \lim_{h\to 0} \frac{3x^2+6xh+3h^2-3x^2}{h} = \lim_{h\to 0} \frac{6xh+3h^2}{h}$

$= \lim_{h\to 0} \frac{h(6x+3h)}{h} = \lim_{h\to 0} 6x+3h$

a) $f'(x) = 6x$

You have 3 "different" definitions of the derivative.

9, 13, 15 ← do these 3 times, one for each def'

b) $f'(x) = 6x$

write the eqn of tangent line at $a=3$

What do I need to accomplish this?

templates

slope $= m$ at $x=a=3$ is $f'(3) = 6(3) = 18$

Point $= (x_0, y_0) = (3, f(3)) = (3, 3(3)^2) = (3, 27)$

slope-intercept form

$y = mx+b$

$y_0 = mx_0+b$

$b = y_0 - mx_0$

point-slope form

$y - y_0 = m(x-x_0)$

$y-27 = 18(x-3)$

find the slope between (x_0, y_0) and (x_1, y_1)

$m = \frac{y_1-y_0}{x_1-x_0} = \frac{\Delta y}{\Delta x}$

find the slope between (x_0, y_0) and (x, y)

$(x-x_0)\left(m = \frac{y-y_0}{x-x_0}\right)(x-x_0)$

3.2 more derivatives 2014-10-07 day 30

26/2 w/w $f(x)=\sqrt{x+1}$; $a=8$

$$f'(x)=\lim_{w\to x}\frac{f(w)-f(x)}{w-x}=\lim_{w\to x}\frac{\sqrt{w+1}-\sqrt{x+1}}{w-x}$$

$$=\lim_{w\to x}\frac{(\sqrt{w+1}-\sqrt{x+1})}{(w-x)}\cdot\frac{(\sqrt{w+1}+\sqrt{x+1})}{\sqrt{w+1}+\sqrt{x+1}}$$

$$=\lim_{w\to x}\frac{(w+1)-(x+1)}{(w-x)(\sqrt{w+1}+\sqrt{x+1})}$$

$$=\lim_{w\to x}\frac{1}{\sqrt{w+1}+\sqrt{x+1}}=\frac{1}{\sqrt{x+1}+\sqrt{x+1}}$$

$$=\frac{1}{2\sqrt{x+1}}$$

$f'(a)=\frac{1}{2\sqrt{a+1}}$

$f'(x)=\frac{1}{2\sqrt{x+1}}$

3.2 more derivatives

2014-10-07 day 30

17) $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ [$\Delta x = x + \Delta x - x$]

$f(x) = ax^2 + b$

$= \lim_{\Delta x \to 0} \frac{[a(x+\Delta x)^2 + b] - [ax^2 + b]}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{a(x^2 + 2x\Delta x + (\Delta x)^2) + b - ax^2 - b}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{ax^2 + 2ax\Delta x + a(\Delta x)^2 + b - ax^2 - b}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{2ax\Delta x + a(\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{2ax + a\Delta x}{1}$

$= \lim_{\Delta x \to 0} 2ax$

$f'(x) = \frac{dy}{dx} = 2ax$

when $x = 3$

$f'(3) = \left.\frac{dy}{dx}\right|_{x=3}$

3.2 more derivatives

2014-10-07 day 30

10) $f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}$ $a = -2$

$= \lim_{w \to x} \frac{w^4 - x^4}{w - x} = \lim_{w \to x} \frac{(w^2 - x^2)(w^2 + x^2)}{w - x}$

$= \lim_{w \to x} \frac{(w - x)(w + x)(w^2 + x^2)}{w - x}$

$= \lim_{w \to x} (w + x)(w^2 + x^2) = (x + x)(x^2 + x^2)$

$= (2x)(2x^2) = 4x^3$